Teleportation of bipartite states using a single entangled pair

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Abstract

A class of quantum protocols to teleport bipartite (entangled) states of two qubits is suggested. Our schemes require a single entangled pair shared by the two parties and the transmission of three bits of classical information, as well as a two-qubit gate with an additional qubit at the receiver' location. Noisy quantum channels are considered and the effects on both the teleportation fidelity and the entanglement of the replica are evaluated.

1 Introduction

Quantum mechanics allows the phenomenon of quantum teleportation. An unknown quantum state is destroyed at a sending place (Alice) while a perfect replica appears at a remote place (Bob) via the combined action of a quantum and a classical channels [1]. Teleportation has been demonstrated for the polarization state of a photon [2], the state of a trapped ion [3] and, in the continuous variable regime, for the set of coherent states of a single-mode radiation field [4]. Perfect quantum teleportation requires a maximally entangled state as quantum channel. Nevertheless, the protocol also works for nonmaximally or even mixed quantum channel [5], with fidelity of quantum teleportation still being better than any classical communication procedure.

Initially, attention was mostly focused to teleportation of single-system quantum states, either of finite-dimensional N-level systems [1,6], or of single-mode continuous variables systems [7,8]. More recently, attention has been devoted to teleportation of states of multipartite (possibly entangled) systems. Notice that the sufficient and necessary conditions to induce entanglement on two remote qubits, by means of their respective linear interactions with a two-mode driving field, has been recently investigated [9]. Direct transmission of an entangled state was considered in a noisy environment [10], and the possibility to copy pure entangled states was studied [11]. A straightforward generalization

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of single-body teleportation [12,13], shows that any state of a N-partite system can be teleported using N maximally-entangled pairs as quantum channel and the transmission of classical information. In addition conclusive teleportation in d-dimensional Hilbert space for partially entangled quantum channel has been formulated [14]. Moreover a scheme to teleport two-particle entangled states has been proposed, using three-particle entangled states, either of GHZ- [15] or W-type [16,17], as quantum channel. These schemes can be also generalized to teleport an unknown three-particle entangled state from a sender to any one of N receivers.

In this paper, we focus our attention to teleportation schemes for bipartite (entangled) states of two qubits. Our goal is to weaken the requirements for the quantum channel, *i.e.* to devise teleportation protocols that work with a reduced amount of entanglement shared between the two parties. As we will see, there is a whole class of protocols realizing this task. In our schemes, an unknown bipartite state can be perfectly teleported, sharing a single EPR pair, once one has at disposal the following ingredients: i) a two-qubit Bell measurements; ii) an Hadamard transformation and a single-qubit measurement at the sender's side; iii) an additional qubit and a set of two-qubit unitary transformations at the receiver's location.

The paper is structured as follows. In Section 2 the simplest example of our class of teleportation protocols is described in details, whereas the general scheme is addressed in Section 3. In Section 4 we analyze how our protocols work with noisy quantum channels, and evaluate teleportation fidelity as well as entanglement of the teleported state. Section 5 closes the paper with some concluding remarks.

2 Teleportation of bipartite states using a single entangled pair

Let us consider the following situation. Alice has at her side an unknown bipartite entangled state of the form

$$|\varphi\rangle_{34} = \alpha|00\rangle_{34} + \beta|11\rangle_{34}\,, \tag{1}$$

and she wants to teleport this state to Bob. We assume that Alice and Bob only share a single EPR pair (see Fig.1)

$$|\phi_+\rangle_{12} = \frac{1}{\sqrt{2}}\left\{|00\rangle_{12} + |11\rangle_{12}\right\} \tag{2}$$

Our protocol works as follows: Alice performs a Bell measurement (BM) on qubits 1 and 3 and, after an Hadamard transformation, measure qubit 4.

Then she sends to Bob the results of both measurements (overall three bits of classical information). Bob introduces the additional qubit 5, and performs a C-not transformation on qubits 2 and 5. Finally, he performs a unitary transformation U_j, $j = 1, ..., 8$ chosen accordingly to the information received from Alice. In this way the initial state of qubits 34 is restored on qubits 25.

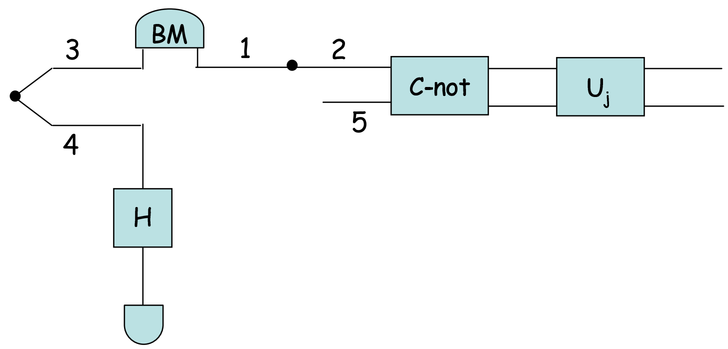

Fig. 1. Schematic diagram of the suggested teleportation protocol. Alice and Bob share a single EPR pair (qubits 1 and 2). Alice performs a Bell measurement (BM) on qubits 1 and 3 and, after an Hadamard transformation (H), measure qubit 4. Then she sends to Bob the results of both measurements (overall three bits of classical information). Bob introduces the additional qubit 5, and performs a C-not transformation on qubits 2 and 5. Finally, he performs a suitable unitary transformation U_j, $j = 1, ..., 8$ according to Alice' results. In this way the initial state of qubits 3 and 4 is restored on qubits 2 and 5.

The initial state of the system is given by

$$|\psi\rangle_{12345} = |\phi_+\rangle_{12}|\varphi\rangle_{34}|0\rangle_5 \,, \tag{3}$$

but, for the moment, let us consider only the first four qubits, which are excited in the state $|\phi_+\rangle_{12}\,|\varphi\rangle_{34}$. Alice performs a Bell measurement on qubits 1 and 3, described by the projectors $\Pi_{\phi_\pm} = |\phi_\pm\rangle_{13\,13}\langle\phi_\pm|$ and $\Pi_{\psi_\pm} = |\psi_\pm\rangle_{13\,13}\langle\psi_\pm|$, with

$$|\phi_\pm\rangle_{13} = \frac{1}{\sqrt{2}}(|00\rangle_{13} \pm |11\rangle_{13}) \qquad |\psi_\pm\rangle_{13} = \frac{1}{\sqrt{2}}(|01\rangle_{13} \pm |10\rangle_{13})$$

According to her results, there are four possible conditional states of qubits 2 and 4, which are given by

$$|\chi_i\rangle_{24} = \alpha|00\rangle_{24} \pm \beta|11\rangle_{24} \qquad i = 1, 2 \tag{4}$$
$$|\chi_i\rangle_{24} = \beta|01\rangle_{24} \pm \alpha|10\rangle_{24} \qquad i = 3, 4\,. \tag{5}$$

The correspondence between the index i and the result of the BM is summarized in Table 1.

Table 1
Summary of the eight possible combinations of Alice's Bell (BM) and qubit 4 (Z_4) results, with the corresponding conditional states of qubits 2 and 4, and conditional transformations (CT) performed by Bob to restore the initial state at his location.

		BM_{13}	CS ϱ_{24}	Z_4	Bob's CT
$j=1$	$i=1$	ϕ_+	$\|\chi_1\rangle_{24}$	0	$\mathbb{I}\otimes\mathbb{I}$
$j=2$	$i=1$	ϕ_+	$\|\chi_1\rangle_{24}$	1	$\sigma_z\otimes\mathbb{I}$
$j=3$	$i=2$	ϕ_-	$\|\chi_2\rangle_{24}$	0	$\sigma_z\otimes\mathbb{I}$
$j=4$	$i=2$	ϕ_-	$\|\chi_2\rangle_{24}$	1	$\mathbb{I}\otimes\mathbb{I}$
$j=5$	$i=3$	ψ_+	$\|\chi_3\rangle_{24}$	0	$\sigma_x\otimes\sigma_x$
$j=6$	$i=3$	ψ_+	$\|\chi_3\rangle_{24}$	1	$\sigma_x\otimes i\sigma_y$
$j=7$	$i=4$	ψ_-	$\|\chi_4\rangle_{24}$	0	$\sigma_x\otimes i\sigma_y$
$j=8$	$i=4$	ψ_-	$\|\chi_4\rangle_{24}$	1	$\sigma_x\otimes\sigma_x$

Let us now take into account also qubit 5 at Bob's location, and assume it is prepared in the state $|0\rangle_5$. At this point, the global conditional state of the three qubits is given by $|\chi_i\rangle_{24}|0\rangle_5$, $i=1,...,4$. We assume that Bob performs a C-not transformation, denoted by the unitary C_{25}, on subsystems 2 and 5, whereas Alice rotates her qubit 4 by a Hadamard transformation. The output states resulting from this procedure are given by $|\phi_i\rangle_{245} = C_{25}\otimes H_4\,|\chi_i\rangle_{24}|0\rangle_5$. Explicitly we have

$$\begin{aligned}
|\phi_1\rangle_{245} &= |0\rangle_4\frac{1}{\sqrt{2}}\left\{\alpha|00\rangle_{25}+\beta|11\rangle_{25}\right\}+|1\rangle_4\frac{1}{\sqrt{2}}\left\{\alpha|00\rangle_{25}-\beta|11\rangle_{25}\right\}\\
|\phi_2\rangle_{245} &= |0\rangle_4\frac{1}{\sqrt{2}}\left\{\alpha|00\rangle_{25}-\beta|11\rangle_{25}\right\}+|1\rangle_4\frac{1}{\sqrt{2}}\left\{\alpha|00\rangle_{25}+\beta|11\rangle_{25}\right\}\\
|\phi_3\rangle_{245} &= |0\rangle_4\frac{1}{\sqrt{2}}\left\{\beta|00\rangle_{25}+\alpha|11\rangle_{25}\right\}-|1\rangle_4\frac{1}{\sqrt{2}}\left\{\beta|00\rangle_{25}-\alpha|11\rangle_{25}\right\}\\
|\phi_4\rangle_{245} &= |0\rangle_4\frac{1}{\sqrt{2}}\left\{\beta|00\rangle_{25}-\alpha|11\rangle_{25}\right\}-|1\rangle_4\frac{1}{\sqrt{2}}\left\{\beta|00\rangle_{25}+\alpha|11\rangle_{25}\right\}\ ,
\end{aligned}\tag{6}$$

where the subscript in $|\phi_i\rangle$ has the same meaning that in Eqs. (4) and (5). Now Alice measures the qubit 4 and sends the results to Bob, together with the result of the BM. The global amount of classical information from Alice to Bob is thus three bits. Indeed, according to the results of the BM and of the measurement of the qubit 4, there are 8 possible conditional states at Bob's site. In order to restore the original state, Bob should perform an appropriate unitary transformation U_j with $j=1,..,8$. It is straightforward to demonstrate that the U_j are the factorized transformations given in Table 1. We have thus demonstrated that teleportation of bipartite states of the form

(1) is possible using a single EPR pair shared between Alice and Bob as far as an additional qubit and a C-not transformation are available at the receiver's location. Three bits of classical information should be sent from Alice to Bob in order to restore the input state.

Notice that the choice of expression (1) for the state to be teleported does not imply loss of generality. In fact, any linear combination of the form

$$|\phi\rangle_{34} = \alpha|m_1, n_1\rangle + \beta|m_2, n_2\rangle \tag{7}$$

with $m_j, n_j = 0, 1$ can be brought to (1) by a unitary transformation at the sender' side and then it can be teleported by the same protocol. Also the choice of the state for the qubit 5 at Bob's location is arbitrary. In fact, any initial preparation $|\theta\rangle_5 = a|0\rangle_5 + b|1\rangle_5$ of qubit 5 can be seen as the action of a given unitary on the state $|0\rangle_5$, *i.e.* $|\theta\rangle_5 = V_5|0\rangle_5$. In this case teleportation can be restored if Bob performs the transformation $C'_{25} = C_{25}\,(\mathbb{I}_2 \otimes V_5^\dagger)$ instead of the C-not. This degree of freedom will be exploited in the next Section, where a whole class of teleportation protocols will be introduced and characterized. Finally, we mention that teleportation works equally well using a different Bell state, either $|\phi_+\rangle_{12}$ or $|\psi_\pm\rangle_{12}$, as quantum channel.

3 Beyond the C-not

Let us consider a teleportation scheme similar to that of Section 2 where, instead of a C-not, Bob is only able to perform a given generic two-qubit transformation F_{25} on qubits 2 and 5 (see Fig. 2). The purpose of this Section is twofold. On one hand, we show that teleportation can be still achieved by *factorized* conditional transformations at Bob site as far as F_{25} ia a nonzero entangling transformation. On the other hand, we show that indeed there exists a class of transformations assuring the success of the protocol, corresponding to a SU(2) symmetry of the set of transformations itself.

We denote the action of F_{25} on the standard basis by $F_{25}|n, m\rangle = |f_{nm}\rangle$. Following the protocol of Section 2, with F_{25} instead of the C-not, we have that after the BM and the measurement of qubit 4, the eight possible conditional states of qubits 2 and 5 are given by

$$\begin{aligned}
|\theta_j\rangle_{25} &= \alpha|f_{00}\rangle_{25} + \beta|f_{11}\rangle_{25} \qquad j = 1, 4 \\
|\theta_j\rangle_{25} &= \alpha|f_{00}\rangle_{25} - \beta|f_{11}\rangle_{25} \qquad j = 2, 3 \\
|\theta_j\rangle_{25} &= \beta|f_{00}\rangle_{25} + \alpha|f_{11}\rangle_{25} \qquad j = 5, 8 \\
|\theta_j\rangle_{25} &= \beta|f_{00}\rangle_{25} - \alpha|f_{11}\rangle_{25} \qquad j = 6, 7\,.
\end{aligned} \tag{8}$$

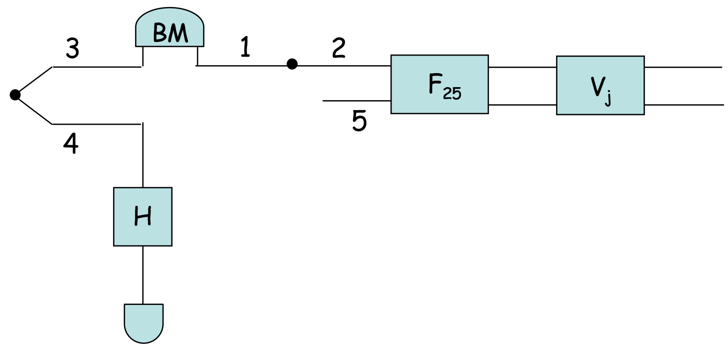

Fig. 2. Schematic diagram of a class of teleportation protocols. Alice and Bob share a single EPR pair (qubit 1 and 2). Alice performs a Bell measurement (BM) on qubit 1 and 3 and, after an Hadamard transformation (H), measure qubit 4. Then she sends to Bob the results of both measurements. Bob performs the two-qubit transformation F_{25} and then the unitary transformation V_j, $j = 1, ..., 8$ according to Alice' results. The initial state of qubits 34 is restored on qubits 25 as far as F_{25} has nonzero entangling power.

Teleportation is possible if we find the appropriate unitaries V_j to restore the initial state. The seeked transformations should fulfill the following requirements

$$\begin{aligned}
&j = 1,4 \quad V_j F_{25}|00\rangle = |00\rangle \quad V_j F_{25}|10\rangle = |11\rangle && (9)\\
&j = 2,3 \quad V_j F_{25}|00\rangle = |00\rangle \quad V_j F_{25}|10\rangle = -|11\rangle && (10)\\
&j = 5,8 \quad V_j F_{25}|00\rangle = |11\rangle \quad V_j F_{25}|10\rangle = |00\rangle && (11)\\
&j = 6,7 \quad V_j F_{25}|00\rangle = |11\rangle \quad V_j F_{25}|10\rangle = -|00\rangle\ . && (12)
\end{aligned}$$

Notice that these relations only determine two lines (the first and the fourth) of the 4×4 matrix representation of each V_j. Eqs. (9-12) thus imply

$$\begin{aligned}
&V_4 F_{25} =^* V_1 F_{25} \qquad V_3 F_{25} =^* V_2 F_{25} && (13)\\
&V_8 F_{25} =^* V_5 F_{25} \qquad V_7 F_{25} =^* V_6 F_{25} && (14)
\end{aligned}$$

where $=^*$ means equality *only* on the relevant lines. Explicitly, Eqs.(9-12) can be rewritten as

$$V_j F_{25} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ z_{j1} & z_{j2} & z_{j3} & z_{j4} \\ z_{j5} & z_{j6} & z_{j7} & z_{j8} \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad V_j F_{25} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ z_{j1} & z_{j2} & z_{j3} & z_{j4} \\ z_{j5} & z_{j6} & z_{j7} & z_{j8} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$V_j F_{25} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ z_{j1} & z_{j2} & z_{j3} & z_{j4} \\ z_{j5} & z_{j6} & z_{j7} & z_{j8} \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad V_j F_{25} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ z_{j1} & z_{j2} & z_{j3} & z_{j4} \\ z_{j5} & z_{j6} & z_{j7} & z_{j8} \\ 1 & 0 & 0 & 0 \end{pmatrix}, \tag{15}$$

where z_{jk} are complex parameters and where, from top left to bottom right $j = 1, 4$, $j = 2, 3$, $j = 5, 8$ and $j = 6, 7$ respectively, Now, imposing the condition of (special) unitarity to matrices in Eq. (15), we obtain the following expression for the second and third line of each product $V_j F_{25}$

$$V_j F_{25} = \begin{pmatrix} .. & .. & .. & .. \\ 0 & \cos\gamma_j & 0 & -\sin\gamma_j \; e^{i(\theta_j - \phi_j)} \\ 0 & \sin\gamma_j \; e^{i\theta_j} & 0 & \cos\gamma_j \; e^{i\phi_j} \\ .. & .. & .. & .. \end{pmatrix}. \tag{16}$$

The initial 8 complex parameter z_{jk} are thus reduced to the three real phases γ_j, θ_j and ϕ_j, corresponding to a SU(2) symmetry of the set V_j. Once the transformation F_{25} is known, Eqs. (15) and (16) fully determine the form the transformations V_j, each of them being determined in a class with three real parameters. This degree of freedom can be used to reduce the number of conditional transformations needed to reconstruct the state nearby Bob, and to realize them as factorized operations on the two qubits. For example, we can choose $V_1 = V_4$, $V_2 = V_3$, $V_5 = V_8$ and $V_6 = V_7$, as it was implicitly assumed in Section 2.

Notice that factorized V_j, as it was in the scheme of Section 2, can be obtained iff F_{25} have a nonzero entangling power *i.e.* full rank, otherwise Eqs. (15) cannot be inverted, and no set of unitaries V_j exists to restore the initial state at Bob site. On the other hand, if we accept that the V_j's may be genuine two-qubit transformations, then (15) have solutions for a generic F_{25}. In this case, however, the role of F_{25} is irrelevant, and it can be included in a redefinition of the V_j's.

4 Effect of noise

Using the protocols described in the previous Sections, with a single pure maximally entangled state as quantum channel, perfect teleportation of bipartite states (1) may be achieved. However, entanglement can be in general corrupted by the interaction with the environment. Therefore, entangled states that are

available for experiments are usually mixed states, and it becomes crucial to establish whether or not the nonlocal character of the protocols has survived the environmental noise.

Teleportation is a linear operation, which also work with mixed states. For a single qubit, quantum teleportation with a mixed quantum channel was studied and it was shown that also when the channel is not maximally entangled the fidelity is better than any classical communication procedure [5].

In this section we study the effect of noise on our schemes, and analyze to which extent the teleportation of bipartite states is degraded. Both fidelity of teleportation and entanglement of the replica will be evaluated. As noisy quantum channel (qubits 1 and 2) we consider a Werner state of the form

$$\rho_c = p|\phi_+\rangle\langle\phi_+| + (1-p)\frac{\mathbb{I}\otimes\mathbb{I}}{4} \; . \tag{17}$$

The same results are obtained using a Werner state built from a different Bell state, either $|\phi_-\rangle$ or $|\psi_\pm\rangle$. As measure of entanglement for the bipartite state ϱ we consider its "negativity"

$$\epsilon = -2\sum_{\lambda_i<0}\lambda_i \; , \tag{18}$$

i.e. the (double) sum of the negative eigenvalues of the partial transpose ϱ^τ. With this definition the quantum channel (17) shows a nonzero degree of entanglement iff $p \geq 1/3$. We have

$$\epsilon_c = \frac{3p-1}{2}; \tag{19}$$

from which we also obtain $p = (2\epsilon_c + 1)/3$. For the state $|\varphi\rangle$ of Eq. (1), *i.e.* the state to be teleported, the negativity is given by

$$\epsilon_\varphi = 2|\alpha|\sqrt{1-|\alpha|^2} \; . \tag{20}$$

Notice that in this case we have $|\alpha|^2 = \frac{1}{2}(1 \pm \sqrt{1-\epsilon_\varphi^2})$ i.e. the same degree of entanglement is obtained for two different values of $|\alpha|^2$ symmetric with respect to $|\alpha|^2 = 1/2$. Using the mixed state (17) as quantum channel our teleportation schemes lead to the following output teleported state for qubits 2 and 5

$$\varrho_t = p|\varphi\rangle\langle\varphi| + (1-p)\frac{\mathbb{I}}{2}\otimes|0\rangle\langle 0|. \tag{21}$$

At this point we calculate the fidelity of teleportation $F = \langle\varphi|\varrho_t|\varphi\rangle$, which is given by

$$F = p + \frac{(1-p)|\alpha|^2}{2} \,, \tag{22}$$

and in terms of the entanglement parameters

$$F_{\pm} = \frac{3\epsilon_c + 1}{4} \pm \frac{\epsilon_c - 1}{12}\sqrt{1-\epsilon_\varphi^2}. \tag{23}$$

In Fig. 3 we show the fidelity as a function of the initial entanglement ϵ_φ for different values of the channel entanglement ϵ_c. Notice that for each value of ϵ_c fidelity shows two branches, corresponding to the two different values of $|\alpha|^2$ that give the same ϵ_φ. By increasing the entanglement of the channel the two branches approach each other, and for $\epsilon_c = 1$ we have $F = 1$ for any value of ϵ_φ. The average fidelity over all possible input states, *i.e.* over all possible values of α is given by

$$\overline{F} = \frac{(1+2p)}{3} \,. \tag{24}$$

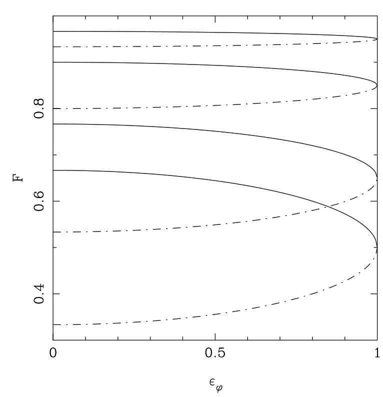

Fig. 3. Fidelity of teleportation as a function of the initial entanglement ϵ_φ for different values of the channel entanglement ϵ_c. The solid and the dot-dashed lines refer to the two branches $F_\pm$ respectively. From top to bottom the curves are plotted for $\epsilon_c = 0,9; 0,7; 0,3; 0$.

In order to evaluate the entanglement of the teleported state ϱ_t we calculated the eigenvalues of the partial transpose. There is a single negative eigenvalue, given by

$$\lambda_{neg} = \frac{1}{4}\left[(1-p) - \sqrt{1 + p[16|\alpha|^2(1-|\alpha|^2)p + p - 2]}\right] \,. \tag{25}$$

Using the above expression together with Eqs. (19) and (20) the entanglement of the replica state can be written as

$$\epsilon_t = \frac{1}{3}\left\{\epsilon_c - 1 + \sqrt{(1-\epsilon_c)^2 + \epsilon_\varphi^2(1+2\epsilon_c)^2}\right\} \ . \tag{26}$$

In Fig. 4 we show the entanglement ϵ_t of the replica as a function of the initial entanglement ϵ_φ for different values of channel entanglement ϵ_c, and as a function of the channel entanglement ϵ_c for different values of the initial entanglement ϵ_φ. Notice that if the entanglement of the initial state ϵ_φ is zero, then the entanglement of the teleported state is always zero, independently on the entanglement of the quantum channel. On the contrary, if ϵ_φ is different from zero we have that ϵ_t is different from zero also when ϵ_c is zero. This is due to the main feature of our schemes, *i.e.* the use of a single pair as quantum channel. In other words, for any entangled $|\varphi\rangle$ the state (21) remains entangled also for the range of values $p < 1/3$.

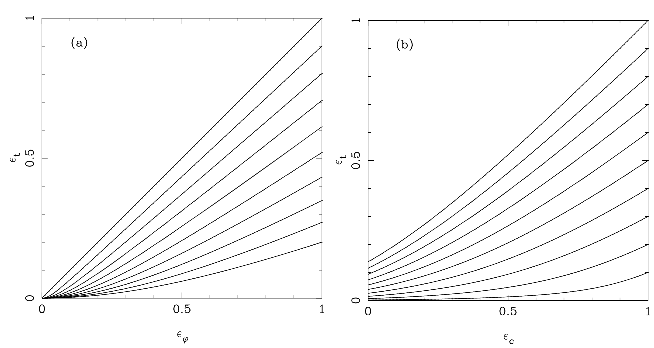

Fig. 4. (a): Entanglement of the teleported state ϵ_t as a function of the initial entanglement ϵ_φ for different values of the channel entanglement ϵ_c from 0 to 1 with a step 0.2. (b): Entanglement of the teleported state ϵ_φ as a function of the channel entanglement ϵ_c for different values of the initial entanglement ϵ_φ from 0 to 1 with a step 0.2.

5 Conclusions

We have suggested a class of quantum protocols to teleport bipartite entangled states of two qubits. Our schemes require a single entangled pair shared by the two parties and the transmission of three bits of classical information. Compared to previous proposals, using two EPR pairs or tripartite entangled states as quantum channels, our schemes require a reduced amount of entanglement to achieve the same task. On the other hand, a larger amount of classical information should be transmitted, and the introduction of an additional qubit and of an entangling transformation nearby Bob is required. Noisy quantum

channels have been considered and the effects on the teleportation fidelity and on the entanglement of the replica have been evaluated.

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